Symbolic AI: Constraint Satisfaction

Google OR Tools: CP-SAT

COMP 741/841 Week 5 - pring 2024

Agenda

- Constraint Satisfaction
- Getting started on Lab 4
- Assigned reading

Constraint Satisfaction Problem (CSP) Example

- **Four-color map coloring theorem**
  - No more than **four colors** are required to color the regions of any map
  - Such that **no two adjancent regions** have the **same color**.
- Map coloring problem example:
  - Color the map of the U.S.
- Map coloring problem can be represented as a CSP
  - How do we represent the US map coloring with constraints?
- Map coloring problem solvin process: CSP search algorithm
  - What CSP algorithm solves the problem?

Representation of a CSP

CSP is represented by 3 components:

- Set of variables
  - Example: 50 variables represent the 50 states
- Domain of values associated with each variable
  - Example: 4 colors (e.g., green, blue, orange, magenta)
- Constraints or relationships among variables
  - Example: no two neighboring states are colored with the same color

Solving a CSP

Assign values to variables such that all constraints are satisfided.

Algorithm: Depth-first search search combined with:

- **Variable order heuristic**
  - Choose state with fewest colors left
  - If a tie, choose state with most uncolored neighbors
- **Inference: forward checking propagation**
  - After assigning a color, propagate the value assignment by
  - Removing that color from the colors of neighboring states

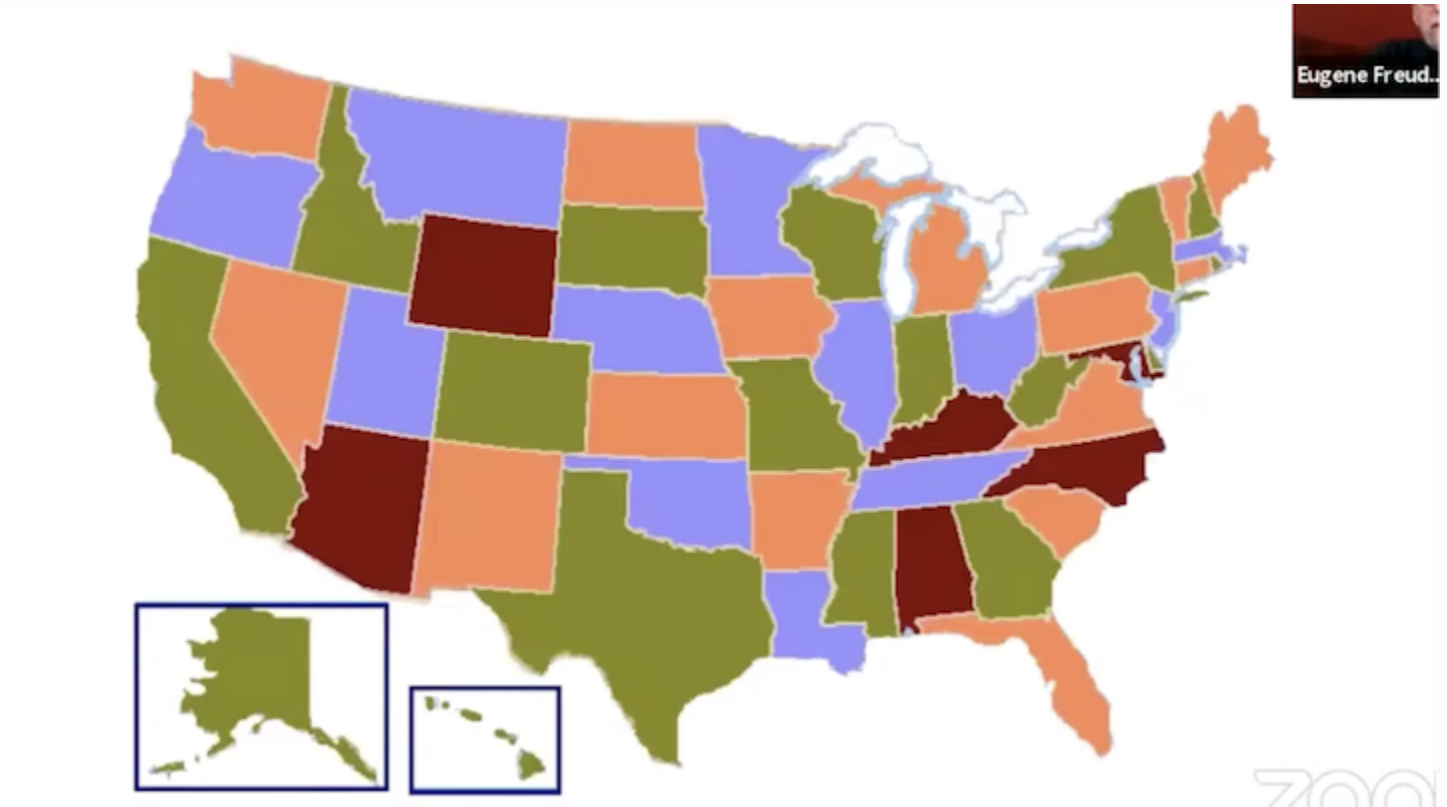
Forward Checking Propagation Example

- Pick Missouri and assign color green
- Propagate the effect: None of Missouri neighbors can be colored green

Variable Ordering Heuristic

- Which state to choose next?
- Rule 1: Choose the state with fewest colors left
- Rule 2: Or, break the tie and choose the state with the most uncolored neighbors

US Map Coloring Solution

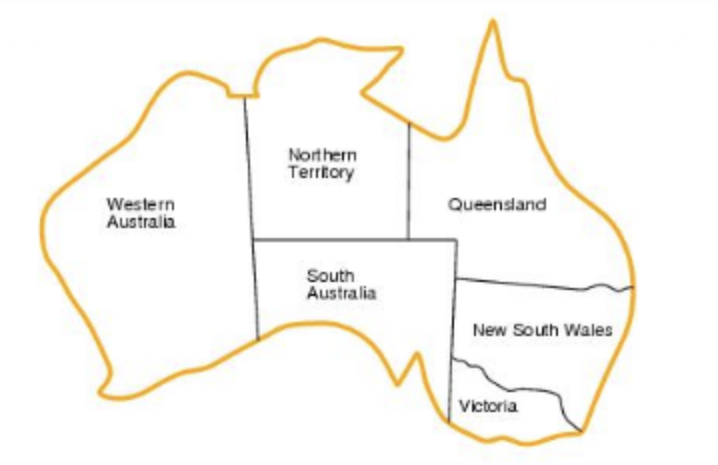


Freuder, Eugene C. 2021. “Ubiquity of Constraints.” IJCAI 2020 Research Excellence Award presented at the International Joint Conference on Artificial Intelligence, Japan, January. https://ijcai20.org/excellence-research-award-session/.

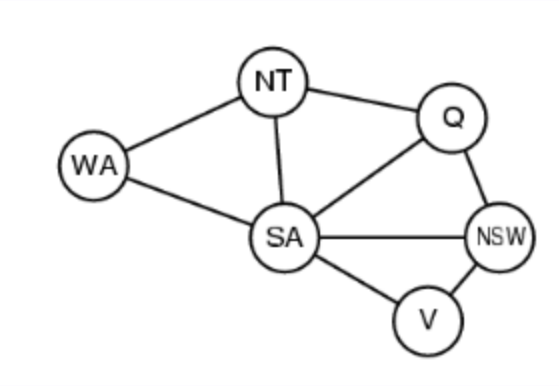
Map Coloring Example

What is the CSP graph of Australia's Mainland states and territories?

- Note: It does not include the state of Tasmania island



Map Coloring CSP Graph Example



- Variables: states, represented by nodes
- Values: 3 colors (reg, green, blue)
- Constraints: binary (between two variables), not equal

Constraint Programming (CP)

- Models problems in terms of *constraints*
- Finds *feasible* solutions from a very large set of *candidate* solutions
  - Feasible solution satisfies all the constraints
  - Candidate solution is partially feasible and **may** satisfy all the constraints.
- A CP problem is a Constraint Satisfaction Problem (CSP)
- CP solutions contain the explanation of the solution. **Why?**

Google OR Tools Library

- Derives its name for **Operations Research (OR)**
  - Discipline that uses math and stats to produce decisions that improve performance, subject to various constraints
    - minimize cost, OR
    - maximize efficiency
- Is used to solves CP problems
- Has TWO CP solvers
  - CP-SAT in `cp_model` module in `ortools.sat` package
  - Original CP in `pywrapcp` in `ortools.constraint_solver`

CP-SAT vs CP-Original

- CP-SAT is more efficent than CP-Original
  - Navigates efficiently a large search space
  - Solves problems for which we don't have efficient algorithms (that have polynomial time efficiency)
  - Brute-force approaches (trying out ALL value assignment to ALL variables) would take too long to prove usable
- CP-SAT is effective for real-world problems: logistics, routing, scheduling
  - Not so much for pseudo-randomly generated problems or or cryptographic problems

Assigned Reading RN3

Freuder, Eugene C. 2021. Ubiquity of Constraints. IJCAI 2020 Research Excellence Award presented at the International Joint Conference on Artificial Intelligence, Japan, January. https://ijcai20.org/excellence-research-award-session/